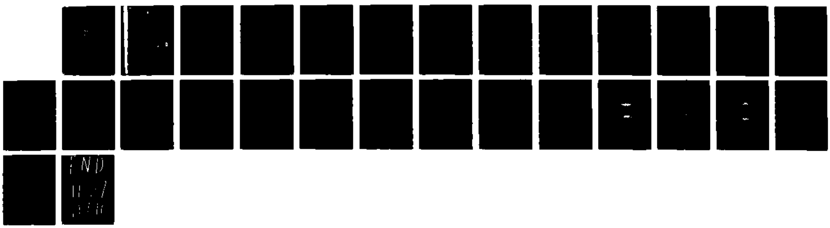
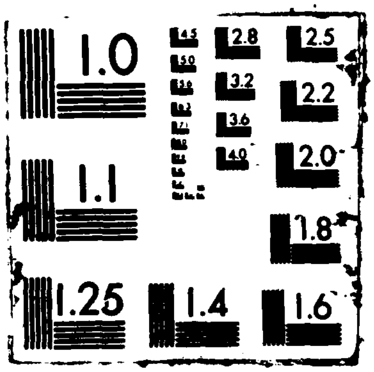
AD-A185 023 2D MAGNETOSTATIC CALCULATIONS AND IMPLICATIONS FOR FERROMAGNETIC MINE CASES(U) MATERIALS RESEARCH LABS ASCOT VALE (AUSTRALIA) P N JOHNSTON MAR 87 MRL-R-1048 1/1

UNCLASSIFIED F/G 17/6 NL




1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

(12)

MRL-R-1048

AR-005-124

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

MATERIALS RESEARCH LABORATORIES

MELBOURNE, VICTORIA

REPORT

MRL-R-1048

2D MAGNETOSTATIC CALCULATIONS AND IMPLICATIONS FOR FERROMAGNETIC MINE CASES

P.N. Johnston

MARCH 1987

DEPARTMENT OF DEFENCE

MATERIALS RESEARCH LABORATORIES

REPORT

MRL-R-1048

2D MAGNETOSTATIC CALCULATIONS AND IMPLICATIONS FOR FERROMAGNETIC MINE CASES

P.N. Johnston

ABSTRACT

In order to assess the effect of a magnetic mine case on a magnetic sensor in or near the case, some 2D magnetostatic calculations have been performed using the boundary element method. For the case of a cylindrical tube the analytic solution to the problem is presented allowing the analytic and numerical solutions to be compared. Conclusions are drawn about the likely effect of a ferromagnetic case on a sensor in or near the case.

Approved for Public Release

POSTAL ADDRESS: Director, Materials Research Laboratories
P.O. Box 50, Ascot Vale, Victoria 3032, Australia

CONTENTS

2D MAGNETOSTATIC CALCULATIONS AND IMPLICATIONS FOR FERROMAGNETIC MINE CASES

1. INTRODUCTION

The presence of a ferromagnetic object in an otherwise uniform magnetic field causes an anomaly (local distortion of the field). The anomaly can be used to signal the presence of a ferromagnetic object e.g. a buried piece of ordnance. A time-varying anomaly can be used to signal the arrival of a vehicle e.g. sensors in the road connected to traffic lights.

The presence of magnetic materials in the vicinity of a magnetic sensor will alter its apparent sensitivity (its real sensitivity is of course constant, but the magnetic field distortion makes it appear changed) to magnetic field changes. This poses a problem when considering a sensor in a mine with a ferromagnetic case. Many currently available mines have this arrangement.

The magnetic sensor of ground mines (sea mines laid on the sea bottom) is usually the sensor used to determine (i) whether the mine is actuated or the ship-count is progressed and (ii) the position along the ship's path relative to the mine at which actuation occurs. Therefore the apparent sensitivity of the magnetic sensor affects the accuracy of target ranging. If the sensitivity of the mine is variable, it cannot be matched to damage radii, thus reducing the mine's effectiveness. Consequently some examination of the effect of ferromagnetic mine cases on the apparent sensitivity of magnetic sensors is necessary.

In this report the effects on sensitivity to an external field of a magnetic sensor placed in or near the wall of a ferromagnetic case are examined. The consequences for the sensitivity of magnetic mines are discussed. The magnetic field problems caused by ferromagnetic mine cases are investigated using analytic and numerical techniques in two dimensions (2D). The use of three-dimensional (3D) calculations, while possible, would involve vastly greater computing resources and the development of codes which may not prove very useful.

2. THEORY OF MAGNETOSTATICS

Using the definitions and conventions of [1], the basic equations of magnetostatics are

$$\nabla . \underset{\sim}{B} = 0 \qquad \nabla \times \underset{\sim}{H} = \frac{4\pi}{c} \underset{\sim}{J} \tag{1}$$

which reduce to

$$\nabla . \underset{\sim}{B} = 0 \qquad \nabla \times \underset{\sim}{H} = \underset{\sim}{0} \tag{2}$$

in the absence of a current density ($\underset{\sim}{J} = \underset{\sim}{0}$). Equation (2) implies that the magnetic field intensity ($\underset{\sim}{H}$) is derivable from a scalar potential ϕ defined by $\underset{\sim}{H} = - \nabla\phi$. Furthermore, for a linear, isotropic material the magnetic induction $\underset{\sim}{B} = \mu\underset{\sim}{H}$ where μ is the magnetic permeability of the material. Then $\nabla . \underset{\sim}{B} = 0$ implies $\nabla . \underset{\sim}{H} = 0$ and therefore ϕ satisfies Laplace's equation

$$\nabla^2 \phi = 0. \tag{3}$$

Laplace's equation is commonly encountered in electrostatics, fluid flow and other physical problems. In 2D, Laplace's equation can be solved analytically for some special cases, including the circular tube case, but in general the solution is obtained numerically.

No problem involving partial differential equations is completely specified without boundary conditions. The problems of interest here involve a ferromagnetic shell (representing a mine case) in an otherwise uniform magnetic field of infinite extent (e.g. the problem of a cylindrical tube of high permeability; Figure 1). Two types of boundary conditions arise naturally from such a problem. Firstly, the field far from the anomaly will be undistorted. In this case $\underset{\sim}{B}$ and $\underset{\sim}{H}$ are uniform and point in the direction of the applied field. The scalar potential ϕ and the field $\underset{\sim}{B}$ are given by

$$\phi = - B_o \, r\cos\theta \qquad \underset{\sim}{B} = B_o \, \hat{\underset{\sim}{x}} \tag{4}$$

where B_o is the magnitude of the uniform field, r and θ define the location of a point in polar coordinates, and $\hat{\underset{\sim}{x}}$ is a unit vector parallel to the uniform field. The other boundary condition of interest in these problems is at a magnetic interface where

$$\underset{\sim}{B}_1 \cdot \underset{\sim}{n}_1 = \underset{\sim}{B}_2 \cdot \underset{\sim}{n}_2$$

$$\underset{\sim}{H}_1 \times \underset{\sim}{n}_1 = \underset{\sim}{H}_2 \times \underset{\sim}{n}_2 \tag{5}$$

irst of these arises from the conservation of magnetic flux across the face i.e. there are no sources or sinks of magnetic flux (magnetic monopoles); while the second arises from the continuity of the tangential component of the magnetic field intensity in the absence of a surface current.

3. LIMITATIONS OF THE PHYSICAL MODEL

In section 2, an approach to magnetostatics problems has been developed which will be applied to specific problems in sections 4, 5 and 6. Before examining these specific problems, the limitations of the approach due to the nature of magnetic materials will be examined.

All the materials under consideration are assumed to have linear and isotropic magnetic properties. Ferromagnetic materials exhibit very large magnetization due to alignment of electron spins in small regions known as domains, which in turn can be collectively oriented. The maximum possible magnetization is obtained when all available electron spins are aligned. This is known as saturation and is the principal cause of non-linear behaviour in magnetic materials. The magnetization of a given sample is not only due to the external field but also to the material's previous history. Thus ferromagnetic materials are highly non-linear and the external field does not completely determine their magnetization. Furthermore, few ferromagnetic materials are isotropic. The equation $\underset{\sim}{B} = \mu \underset{\sim}{H}$ defines μ rather than being a material constant relating $\underset{\sim}{B}$ and $\underset{\sim}{H}$.

The assumptions in section 2 obviously limit the usefulness of performing simple magnetostatic calculations. However, a mine case in the earth's magnetic field is very unlikely to be subjected to an external magnetic field strong enough to cause the case material to approach magnetic saturation. The normal situation of interest involves a small transient change to a larger, effectively constant geomagnetic field. The assumption of linear response to a small transient will be reasonable in many circumstances.

As already discussed permeability is not a well defined quantity for ferromagnetic materials. The incremental permeability at a given point on the hysteresis curve is needed for a given problem. In general, this is unknown; however usually some estimate or upper bound is available.

Given these apparent limitations, one can only hope to obtain indicative information about the field in and around a ferromagnetic object without extraordinary information about the magnetic history and state of that object.

4. ANALYTIC SOLUTION OF CIRCULAR TUBE CASE

An analytical solution of the circular tube case (Figure 1) is presented permitting a test of numerical solutions.

The solution of Laplace's equation for a cylindrical problem can be solved using any complete set of functions. Following [1], for a cylindrical problem the natural solution for the potential is of the form

$$\phi = \sum_{\ell=0}^{\infty} \left(a_\ell r^\ell + b_\ell \frac{1}{r^{\ell+1}}\right) P_\ell(\cos\theta) \qquad (6)$$

where (a_ℓ) and (b_ℓ) define the solution in a region and the $P_\ell(\cos\theta)$ are Legendre polynomials. Naturally, as r->0 the terms of the form $r^{-(\ell+1)}$ diverge. In the limit $r \to \infty$ the the non-vanishing terms must tend to the solution for a uniform magnetic field i.e. $\phi = - B_0 r\cos\theta$.
This is essentially the boundary condition far from the anomaly source (4). Thus the solution must be of the form:-

$$\phi = \begin{cases} - B_0 r\cos\theta + \sum_{\ell=0}^{\infty} \frac{\alpha_\ell}{r^{\ell+1}} P_\ell(\cos\theta) & r > b \\ \sum_{\ell=0}^{\infty} \left(\beta_\ell r^\ell + \gamma_\ell \frac{1}{r^{\ell+1}}\right) P_\ell(\cos\theta) & a<r<b \\ \sum_{\ell=0}^{\infty} \delta_\ell r^\ell P_\ell(\cos\theta) & r < a \end{cases} \qquad (7)$$

where (α_ℓ), (β_ℓ), (γ_ℓ) and (δ_ℓ) are determined by the boundary conditions at the magnetic interfaces (5). These boundary conditions mean that only terms with $\ell=1$ can be non-zero. Thus we can drop the summations over ℓ and the subscripts ℓ.

From (5) the boundary conditions at the magnetic interfaces (i.e. at r=a and r=b) give:-

$$- b^3 B_0 = 2\alpha + \mu b^3 \beta - 2\mu\gamma$$

$$0 = \mu a^3\beta - 2\mu\gamma - a^3\delta$$

$$b^3 B_0 = \alpha - \beta b^3 - \gamma \qquad (8)$$

$$0 = a^3\beta + \gamma - a^3 \delta$$

which can be solved to give

$$\alpha = [\frac{(2\mu + 1)(\mu - 1)}{(2\mu + 1)(\mu + 2) - 2(a^3/b^3)(\mu - 1)^2}](b^3 - a^3)B_o$$

$$\beta = [\frac{-3(2\mu + 1)}{(2\mu + 1)(\mu + 2) - 2(a^3/b^3)(\mu - 1)^2}]B_o \qquad (9)$$

$$\gamma = [\frac{-3(\mu - 1)}{(2\mu + 1)(\mu + 2) - 2(a^3/b^3)(\mu - 1)^2}]a^3 B_o$$

$$\delta = [\frac{-9\mu}{(2\mu + 1)(\mu + 2) - 2(a^3/b^3)(\mu - 1)^2}]B_o$$

which are purely functions of geometry, μ and the external uniform field. The magnetic field $\underset{\sim}{H} = -\nabla\phi$ is readily obtained from equations (7) and (9) as;

$$\underset{\sim}{H} = \begin{cases} (-\delta, 0) & r<a \\ (-\beta - \frac{\gamma(y^2-2x^2)}{r^5}, \frac{3\gamma xy}{r^5}), & a<r<b \\ (B_o - \frac{\alpha(y^2 - 2x^2)}{r^5}, \frac{3\alpha xy}{r^5}) & r>b \end{cases} \qquad (10)$$

where the fields have been transformed from cylindrical polar coordinates into rectilinear coordinates with $x = r\cos\theta$ and $y = r\sin\theta$.

The magnetic lines of force around the circular tube computed from the analytic solution are shown in Figure 2 for a=1, b=2 and μ=1000. The value μ=1000 is chosen because it is a value typical of iron-like substances [2]. Figure 2 shows that there is a strong concentration of flux through the ferromagnetic material with the highest concentration being along the axis perpendicular to the direction of the external field (the perpendicular axis). The magnetic lines of force are almost perpendicular to the magnetic interface from outside at r=b. In the limit $\mu \to \infty$ they would be

perpendicular. The concentration of flux through the ferromagnetic material causes increases in the field strength outside the case on the magnetic axis and a decrease along the perpendicular axis. These distortions of the field outside the case constitute the magnetic anomaly and have short range, dropping off as $1/r^3$ from equations (10). Inside the case wall is a highly shielded region.

5. NUMERICAL SOLUTIONS

There are two commonly used numerical methods for the solution of second order partial differential equations; the finite element method and the boundary element method.

In view of the lower complexity and lesser computational power required, the boundary element method is used to examine some of the features of the magnetic field in and near ferromagnetic objects.

A 2D boundary element code written in FORTRAN by Chang [3] from the University of Southampton was used. The code was developed to examine the seepage of water through rocks of differing permeabilities under a given head of pressure. This problem is analogous to the "flow" of magnetic flux through materials of differing magnetic permeability under given initial values of ϕ at some boundaries. The only changes required to the program to run on the MRL VAX 11/780 were file input/output statements. In order to get best use and utility from the code, preprocessing and postprocessing programs have been written which simplify the entry of data to the boundary element program and allow various types of data output. The preprocessing program enabled the rapid generation of the input file, which contains information for each boundary element, from a brief specification of the parameters of each boundary line. The postprocessing program was used mainly for the production of diagrams of magnetic lines of force. Magnetic lines of force are analogous to streamlines for the water seepage problem. These diagrams provide a compact graphical representation of the general characteristics of the magnetic field in a region.

6. RESULTS

In order to assess the value of numerical solutions to the 2D magnetostatic problems under study, the results from the analytical solution of the cylindrical tube problem (section 4) were compared with those from the boundary element method outlined in section 5.

Numerical solutions were obtained using 45,89,178 and 338 boundary elements. The inner radius of the tube was 1 unit, the outer radius 2 units and the extremity of the region considered was 10 units except when 338 boundary elements were used when this was increased to 20 units. The

arrangement of boundary elements for the numerical solution using 178 boundary elements is shown in Figure 3. All elements were made to have approximately the same length.

The numerical solutions indicate similar gross structure to the analytic solution which can be seen from the diagrams showing magnetic lines of force (Figures 2&4).

Figures 5-10 show comparisons of the analytic solution with the various numerical solutions using differing numbers of boundary elements. The fine structure in the analytic solution, e.g. the increase in $\underline{H}\cdot\underline{x}$ near the tube (Figure 6), is not reproduced by the numerical solutions. This feature is common to all of the numerical solutions. Furthermore, the numerical solutions do not asymptote to the analytic solution with increasing numbers of boundary elements. However there are edge or boundary effects with the boundary element method which are related to the size of the elements. Increasing the number of boundary elements allows solutions to be obtained nearer the boundary surfaces.

The implicit averaging in the boundary element method seems to wash out fine structure. This does not pose a great problem considering the inherent limitations of the physical model (section 3). The boundary element method provides the directions of streamlines and estimates of the field strength both inside the material and in the tube wall.

Fortunately the poor results near the boundary surfaces are not important in the current examination of mine cases. However if one wished to perform calculations on a comparatively thin-walled vessel like a real mine case, the boundary effects would mean that many elements would be needed, greatly increasing the cost of the computation.

Figures 11-14 show magnetic lines of force for simple 2D ferromagnetic objects of permeability $\mu = 1000$ in an otherwise uniform magnetic field. These objects were modelled in a region of side length 10 units with boundary elements of length as close as possible to those in the cylindrical tube example using 178 boundary elements (Figure 3). All streamline diagrams are at the same scale.

The anomaly properties outside the object are generally similar to the cylindrical tube example with an increase in the magnetic field strength along the axis of the external field and a decrease along the perpendicular axis. In the case wall the the concentration of flux again has similar properties with greatest concentration along the perpendicular axis. Inside the objects the field is greatly diminished due to shielding.

For the elliptical cases (Figures 14 and 15) these streamline diagrams show that there is a considerable difference in sensitivity for mines aligned with their major and minor axes parallel to the external uniform field (ignoring orientation effects e.g. using a three-axis magnetometer).

7. CONCLUSIONS

Although these calculations were performed in 2D for simple 2D objects, the inferences drawn should be capable of extension to 3D. The diagrams showing magnetic lines of force clearly indicate that the apparent sensitivity of a magnetic sensor will be greatly decreased if it is placed inside a ferromagnetic case. If the sensor is placed in the wall of a ferromagnetic case the sensitivity of the mine will depend on the orientation of the case relative to the external field.

Variability in the apparent sensitivity of a mine sensor makes its use much more difficult in that:-

(a) The sensor response to a given target cannot be accurately specified. Consequently valid targets may be missed in some orientations and damage radii cannot be properly matched to target signal with a consequent reduction in the effectiveness of one's mines.

(b) Difficulty of clearing one's own minefields is increased due to the need to use worst case values for sensitivity which may be unrealistic.

Apart from these operational problems, the magnetic anomaly from a ferromagnetic case provides the enemy with a means of detecting a mine that could be denied him if a non-magnetic case were used.

The boundary element method is capable of being used to determine relative sensitivity of magnetic sensors inside mines of different geometries given that other parameters (e.g. orientation, magnetic history of material) are unchanged. A knowledge of the maximum sensitivity of a mine is necessary for mine clearance operations. An estimate of the maximum sensitivity of a mine may be obtained from a boundary element calculation.

The actual sensitivity of a given mine cannot be predicted if the sensor is in the wall of a ferromagnetic case. To put a sensor inside the case means that a higher sensitivity, more expensive sensor is required due to the shielding effect of a ferromagnetic case. The use of a ferromagnetic mine case means that variability of sensor response and its associated problems are accepted by the designer of the mine.

Clearly, in the design of a mine utilising a magnetic sensor, a non-magnetic case and a three-axis or total field magnetometer would eliminate all orientation and sensitivity of magnetic sensor difficulties.

8. ACKNOWLEDGEMENTS

I would like to thank Drs D.D.Richardson and P.J. Ryan for their assistance and encouragement in this work, and Ms. L. Montelli for her help in preparing the illustrations.

9. REFERENCES

1. Jackson, J.D. (1975). "Classical Electrodynamics", second edition (John Wiley and Sons, New York).

2. Bozorth, R.M. (1951). "Ferromagnetism" (Van Nostrand, New York).

3. Chang, O.V. (1979). "Boundary Elements Applied to Seepage Problems in Zoned Anisotropic Soils", M.Sc. Thesis, University of Southampton (Unpublished).

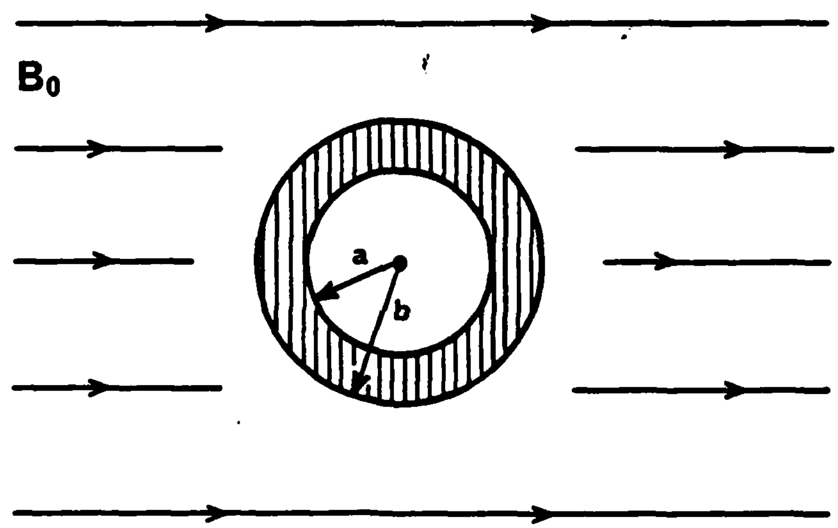


FIGURE 1 Cylindrical tube of high permeability in a region of otherwise uniform magnetic field.

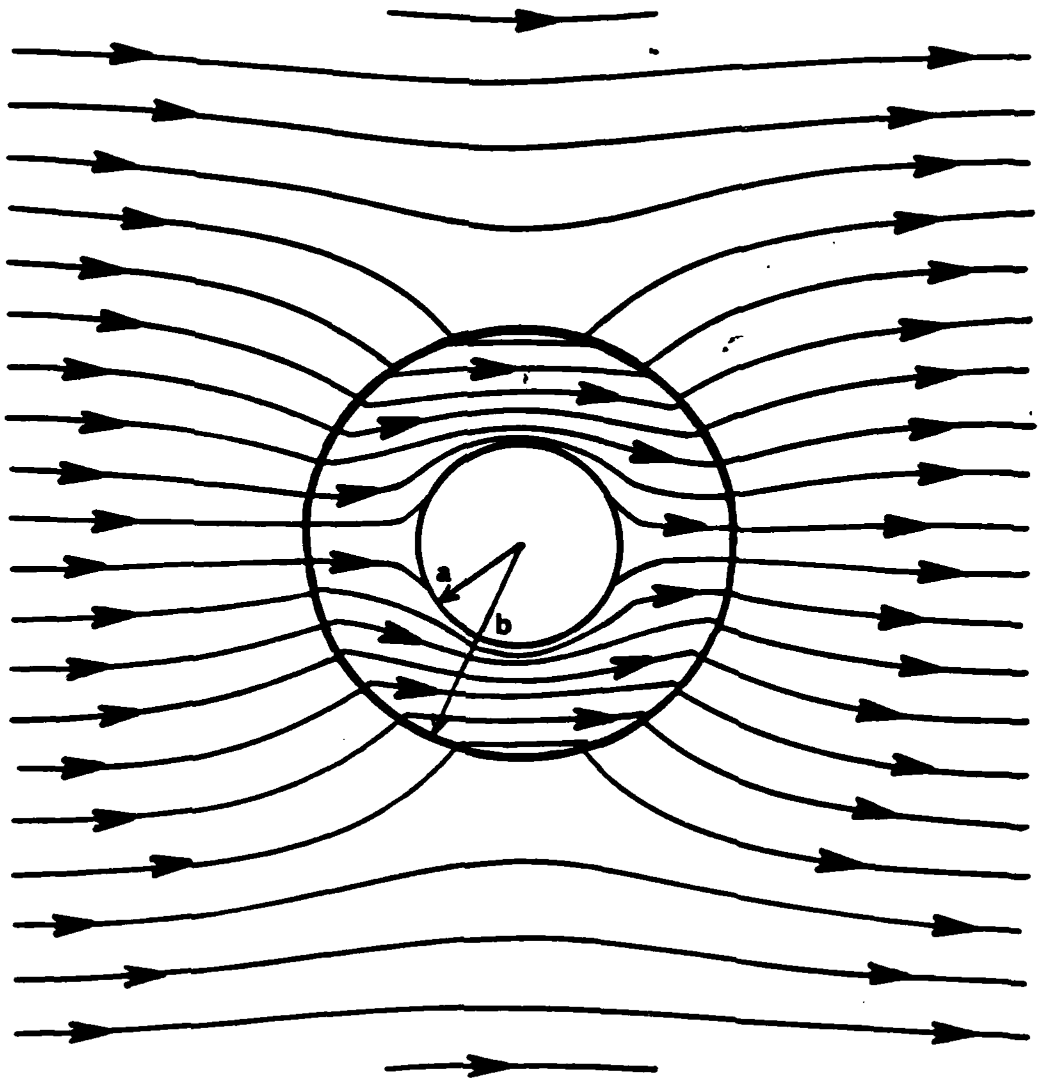


FIGURE 2 Lines of force derived from the analytic solution of the cylindrical tube problem.

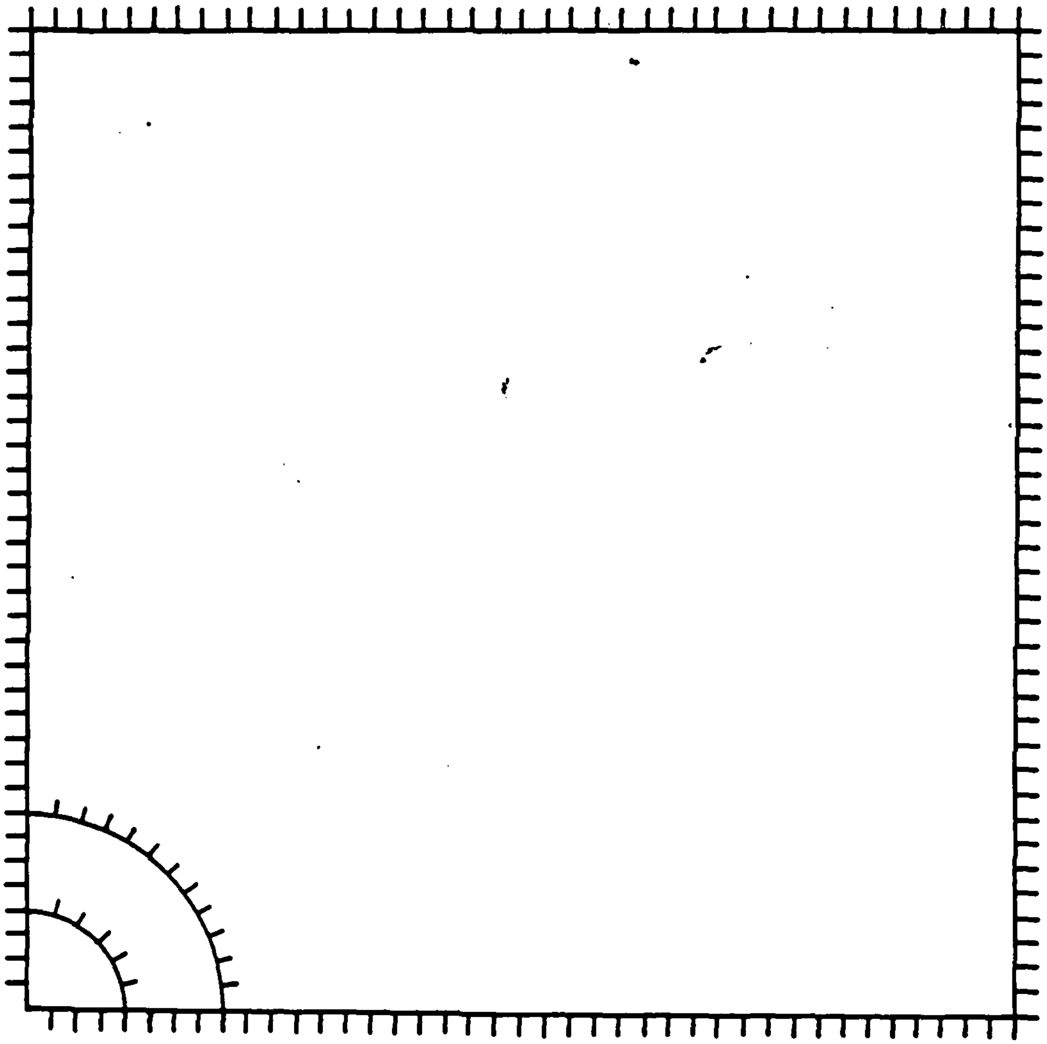


FIGURE 3 The layout of boundary elements (b.e) for the cylindrical tube problem using 178 b.e.

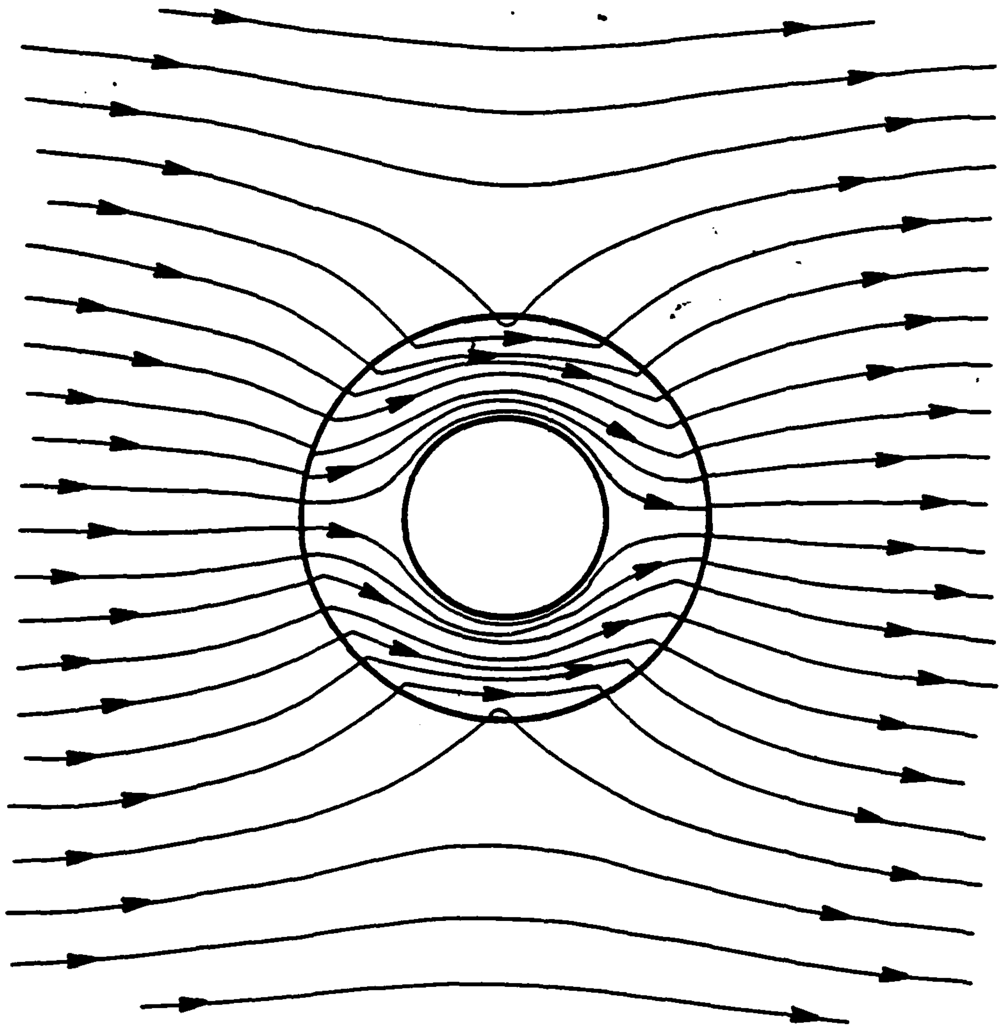


FIGURE 4 Lines of force derived from the 178 b.e numerical solution of the cylindrical tube problem.

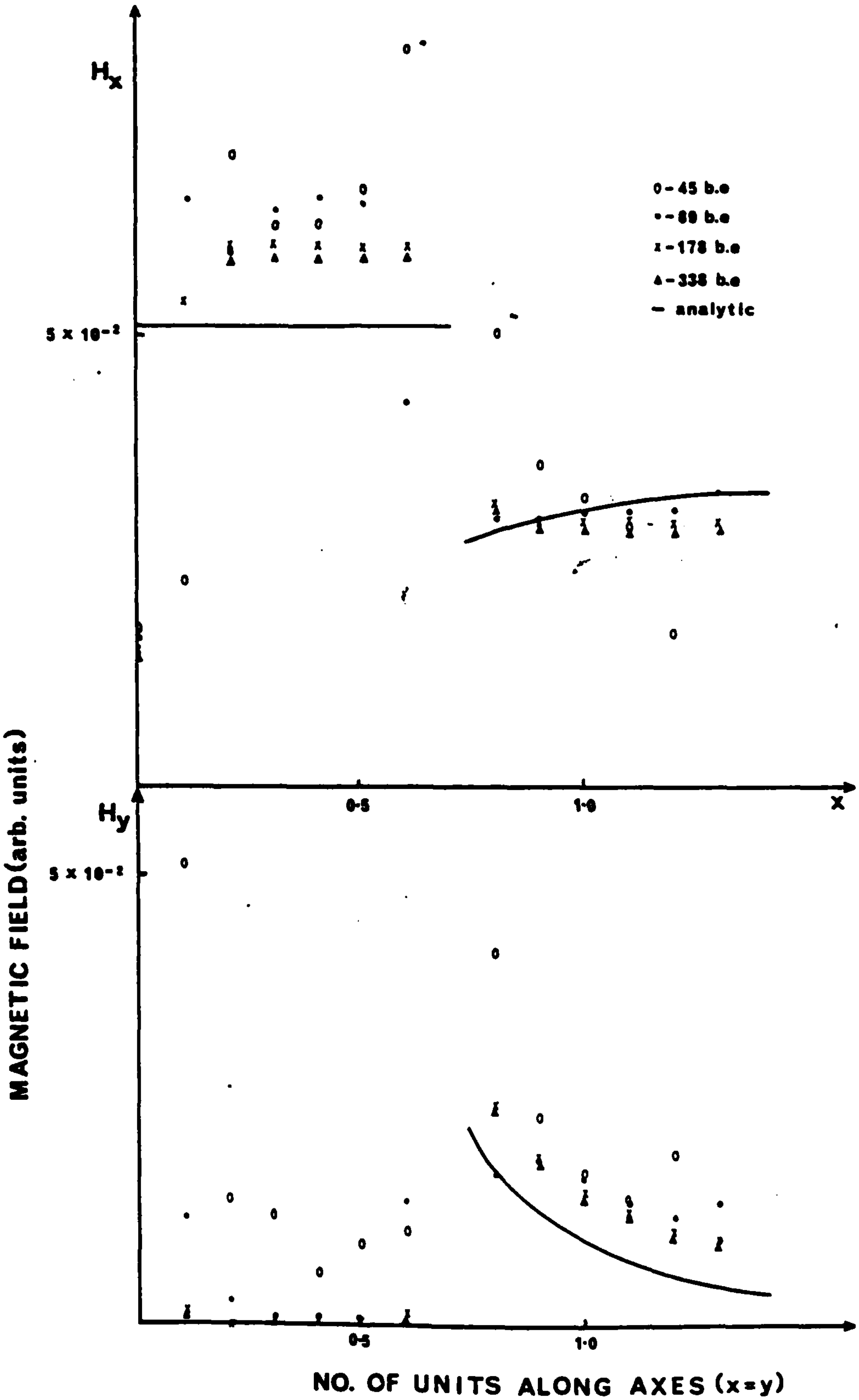


FIGURE 5 Comparison of numerical and analytic solutions of the cylindrical tube problem along the line x=y, inside and in the wall of the tube.

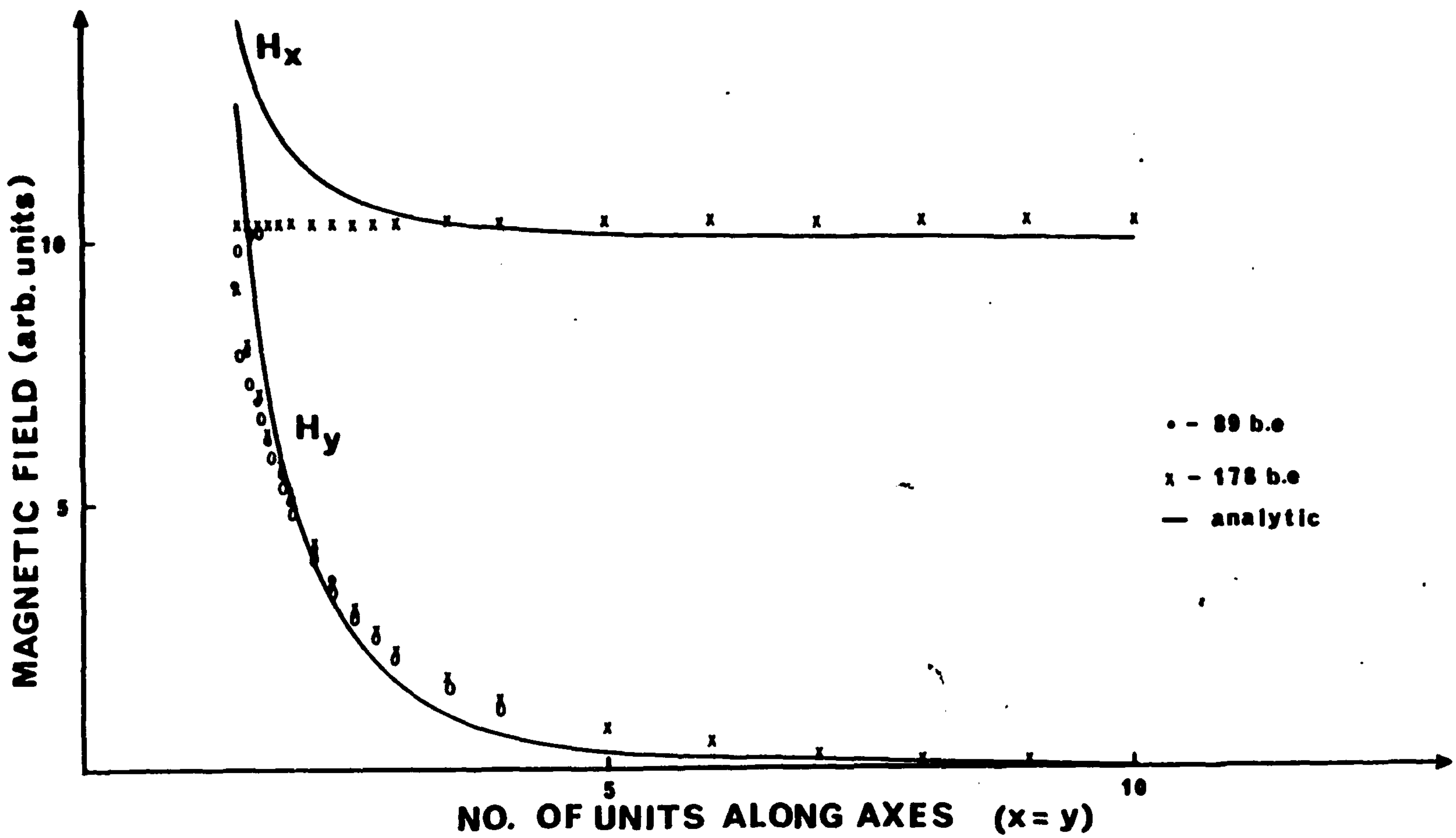


FIGURE 6 Comparison of numerical and analytic solutions of the cylindrical tube problem along the line x-y, outside the tube.

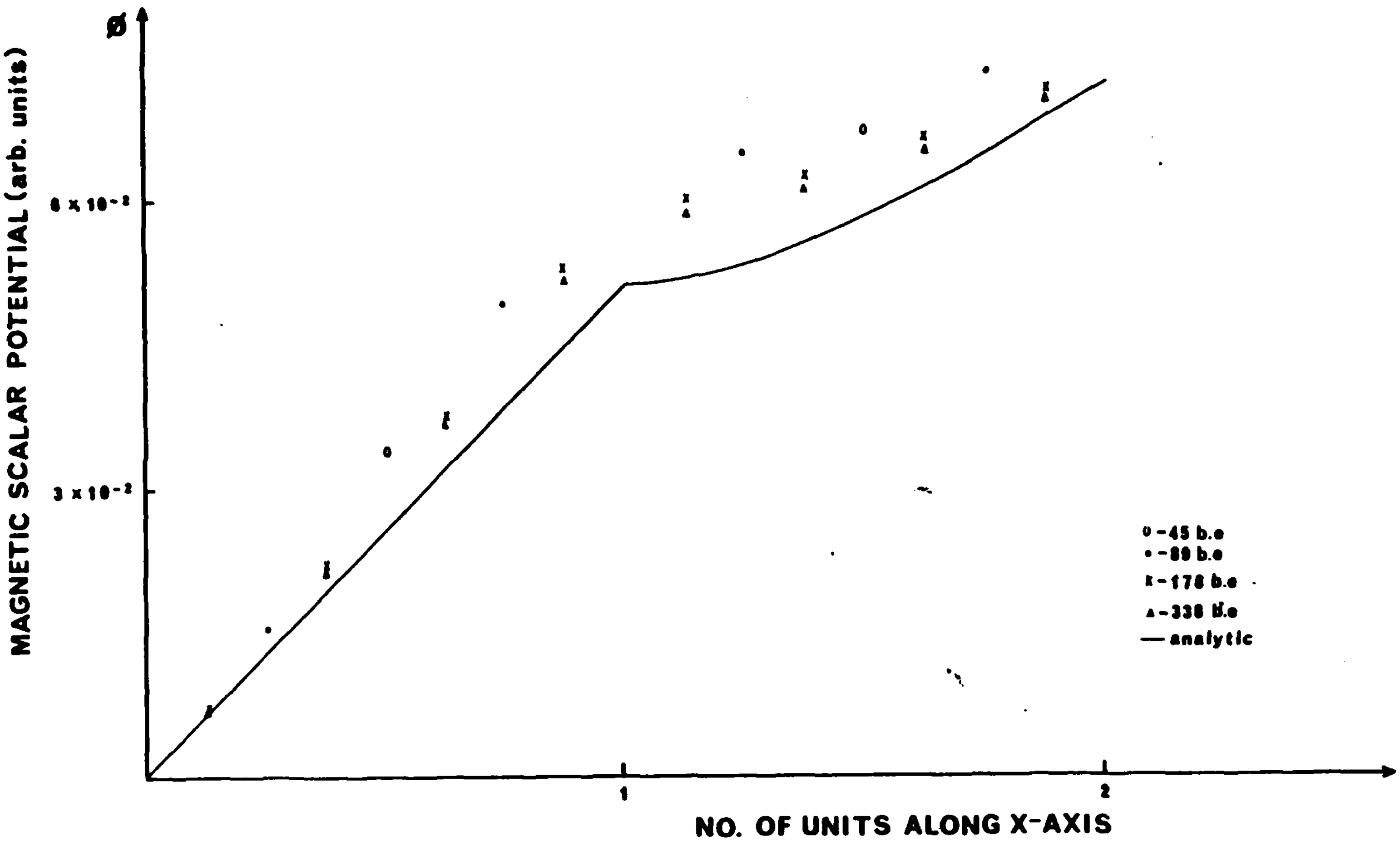


FIGURE 7 Comparison of numerical and analytic solutions for the cylindrical tube problem along the axis of the field through the centre of the tube, inside and in the wall of the tube.

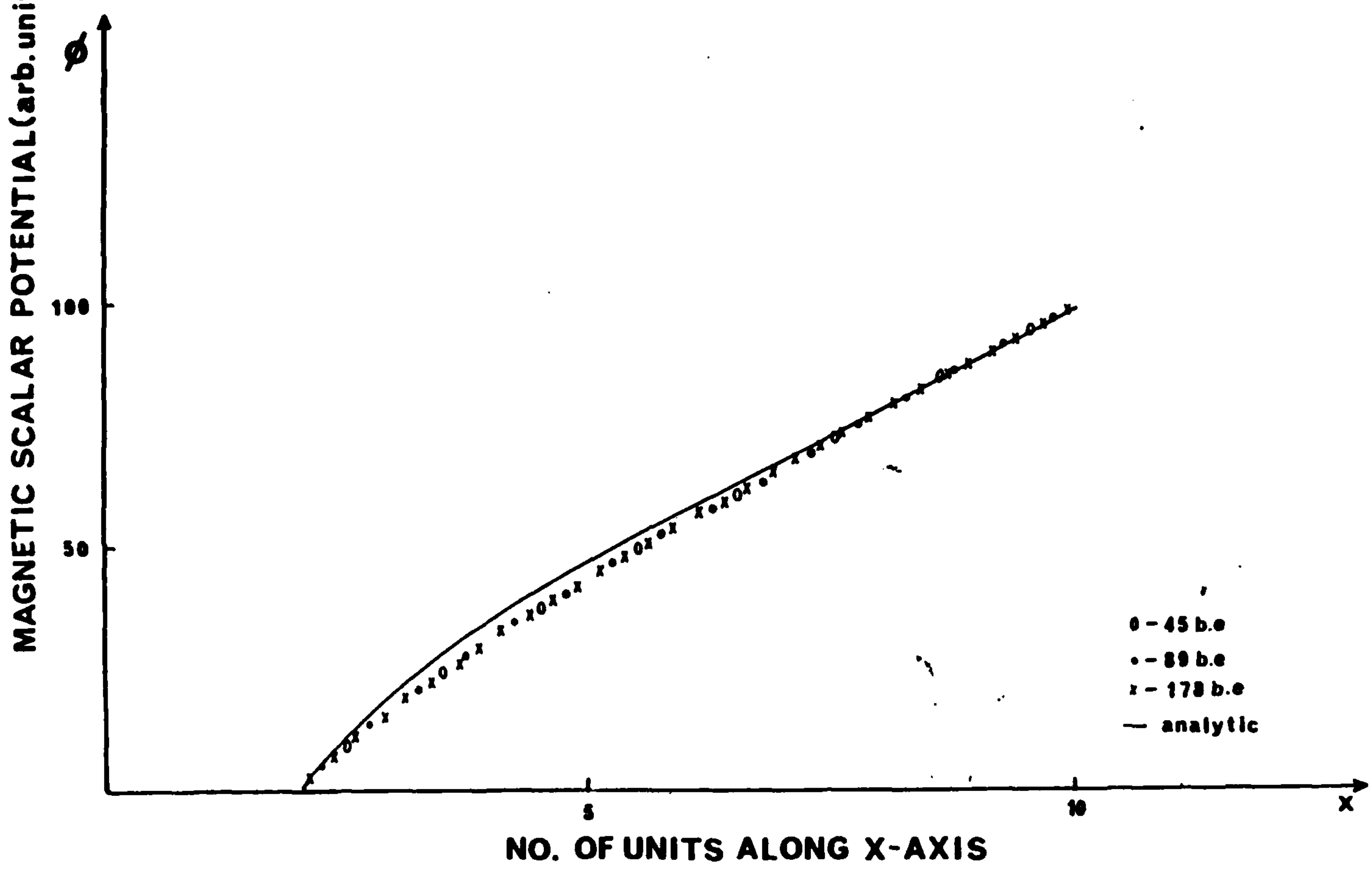


FIGURE 8 Comparison of numerical and analytic solutions for the cylindrical tube problem along the axis of the field through the centre of the tube, outside the tube.

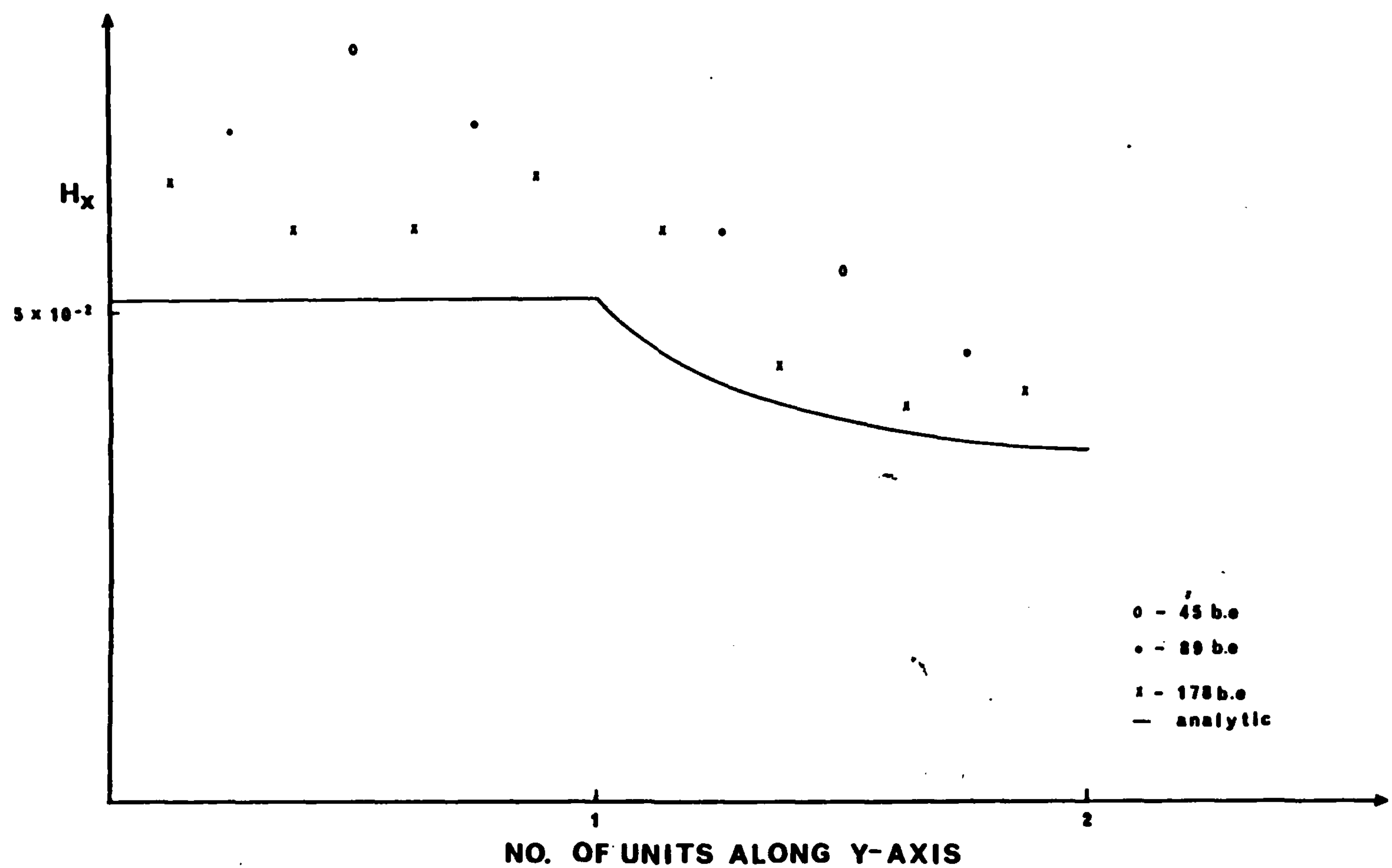


FIGURE 9 Comparison of numerical and analytic solutions of the cylindrical tube problem along the axis of the tube perpendicular to the field, inside and in the wall of the tube.

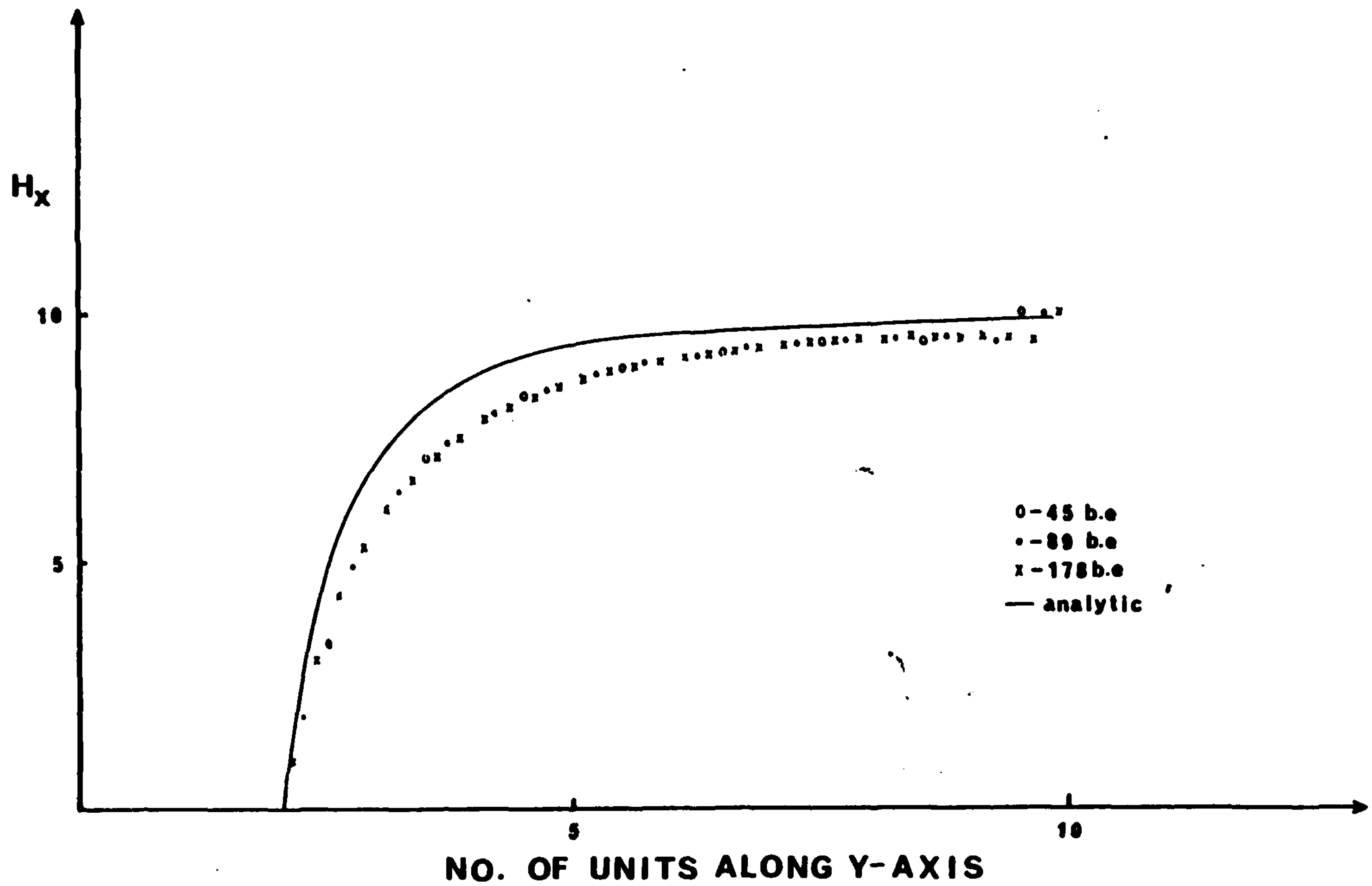


FIGURE 10 Comparison of numerical and analytic solutions of the cylindrical tube problem along the axis of the tube perpendicular to the field, outside the tube.

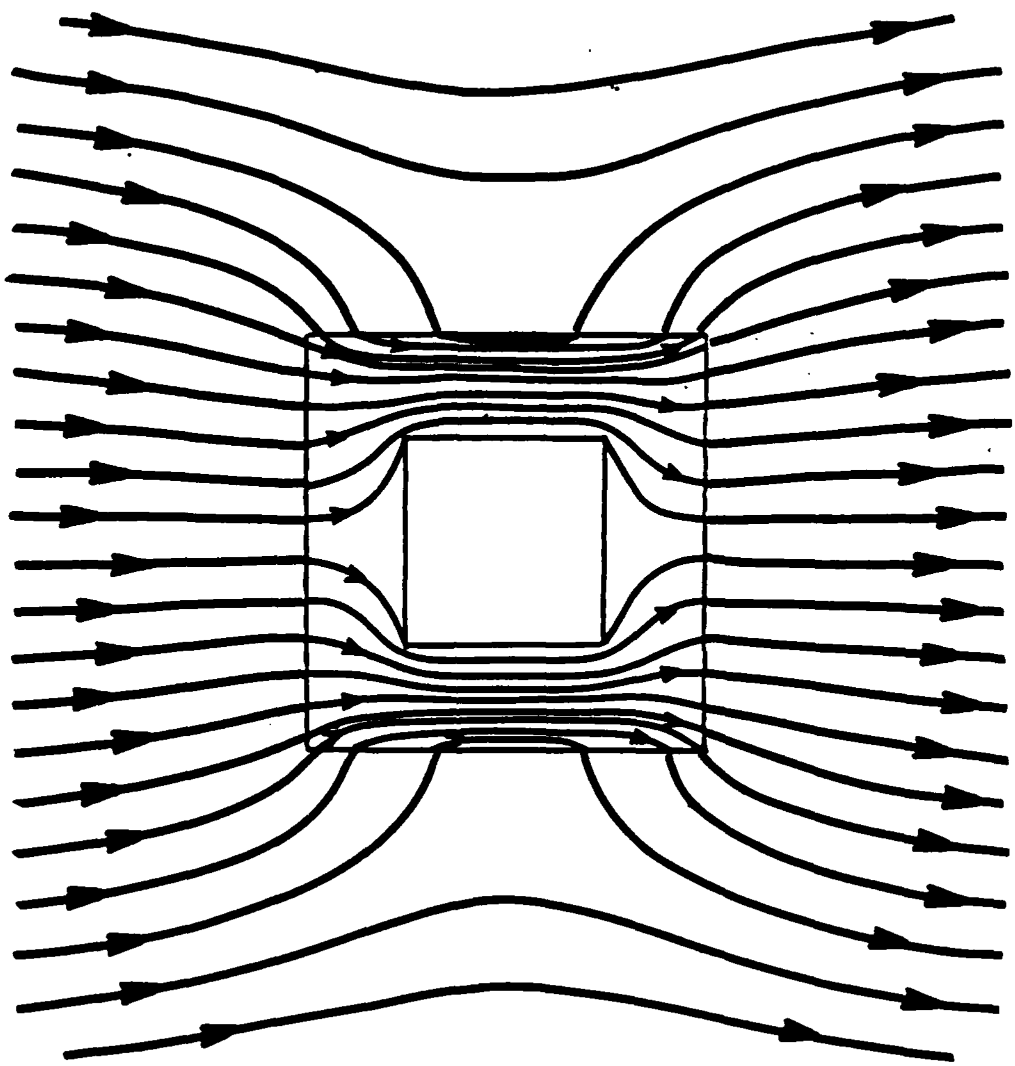


FIGURE 11 Lines of force derived from the 184 b.e numerical solution of the square cross-section tube problem, where the tube is aligned with the field.

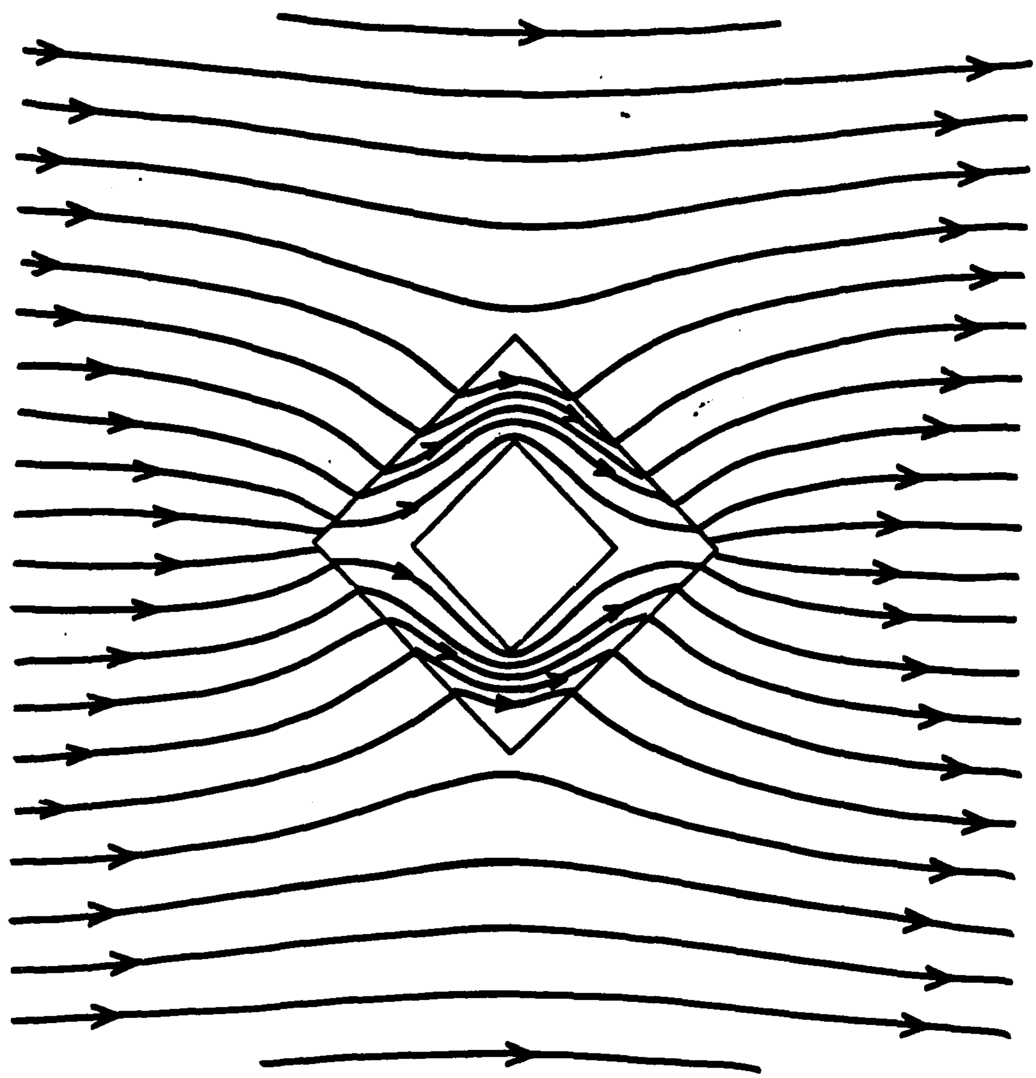


FIGURE 12 Lines of force derived from the 178 b.e numerical solution of the square cross-section tube problem, where the tube is aligned at 45° to the field.

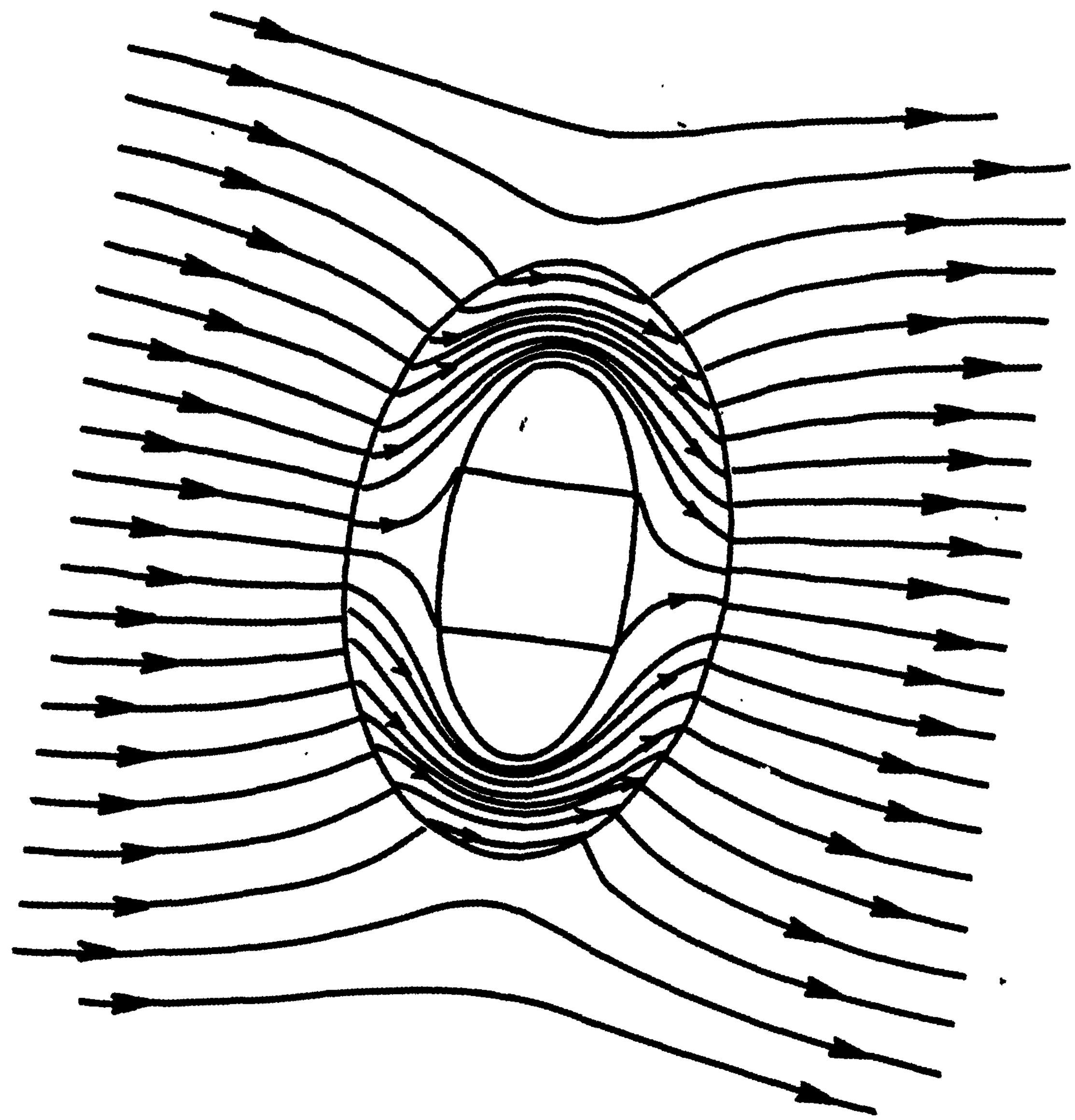


FIGURE 13 Lines of force derived from the 184 b.e numerical solution of the elliptical cross-section tube problem, where the minor axis of the tube is aligned with the field.

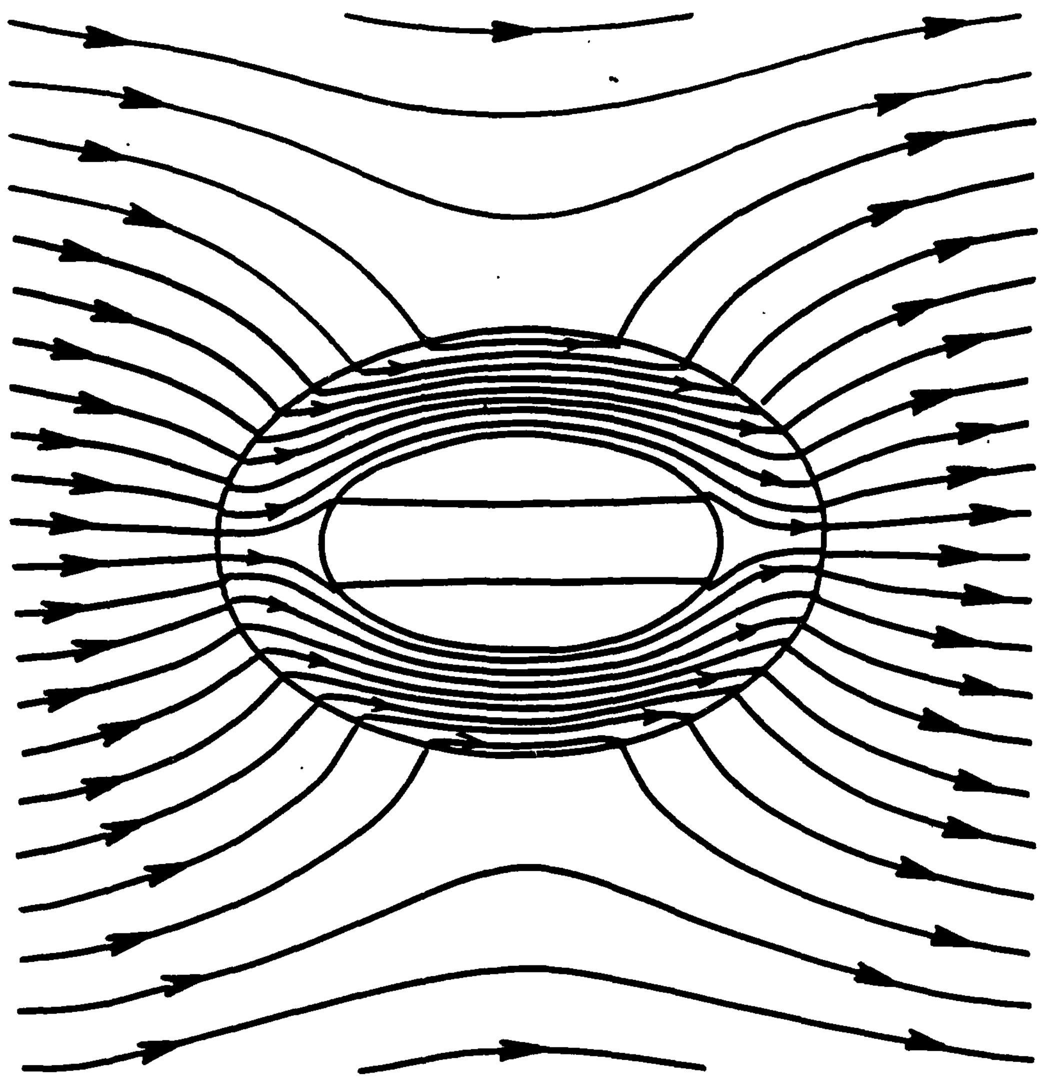


FIGURE 14 Lines of force derived from the 184 b.e numerical solution of the elliptical cross-section tube problem, where the major axis of the tube is aligned with the field.

SECURITY CLASSIFICATION OF THIS PAGE **UNCLASSIFIED**

DOCUMENT CONTROL DATA SHEET

REPORT NO.	AR NO.	REPORT SECURITY CLASSIFICATION
MRL-R-1048	AR-005-124	Unclassified

TITLE

2D Magnetostatic calculations and implications for ferromagnetic mine cases

AUTHOR(S)

P.N. Johnston

CORPORATE AUTHOR

Materials Research Laboratories
PO Box 50,
Ascot Vale, Victoria 3032

REPORT DATE	TASK NO.	SPONSOR
MARCH 1987	DST 85/153	DSTO

FILE NO.	REFERENCES	PAGES
G6/4/8-3310	3	26

CLASSIFICATION/LIMITATION REVIEW DATE	CLASSIFICATION/RELEASE AUTHORITY
February 1990	Superintendent, MRL Physical Chemistry Division

SECONDARY DISTRIBUTION

Approved for Public Release

ANNOUNCEMENT

Announcement of this report is unlimited.

KEYWORDS

Magnetic mines	Sensors	Sensitivity
Magnetic detection	Mine cases	

COSATI GROUPS 0063D 0079I

ABSTRACT

In order to assess the effect of a magnetic mine case on a magnetic sensor in or near the case, some 2D magnetostatic calculations have been performed using the boundary element method. For the case of a cylindrical tube the analytic solution to the problem is presented allowing the analytic and numerical solutions to be compared. Conclusions are drawn about the likely effect of a ferromagnetic case on a sensor in or near the case.

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

END
11-87
DTIC